THEORETICAL STUDY OF UNDULATOR INDUCED TRANSPARENCY IN MAGNETOACTIVE PLASMA

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The self-consistent structure of normal waves is investigated in the regime of undulator induced transparency in magnetized plasma. The mode conversion during the propagation in inhomogeneous plasma is investigated and the conversion efficiency is estimated. The proper profiles of plasma density and external magnetic field are proposed and numerical calculations are used for modelling this effect and the detailed comparison of the results with the theory is performed.

1. Introduction

The effect of electromagnetically induced transparency (EIT), firstly discovered in quantum systems [1] is actively investigated in classical systems (magnetized plasma) now [2-5]. The basic features of EIT are the suppression of resonant absorption of low-power probe wave and strong group velocity deceleration within this “transparency window” with the presence of a strong, appropriately detuned pump electromagnetic wave. The electrostatic plasma oscillations are effectively exited in EIT regime by the beating between probe and pump waves.

In [3,5] it was proposed to use the magnetic undulator as the pump. The undulator coupling yields ultra-slow hybrid electromagnetic waves with group velocity, which is substantially less than the speed of light in vacuum, and leads to the extreme energy compression in the plasma. One of the possible applications of this effect can be ion acceleration, which is accomplished by the longitudinal electric field of the plasma wave.

In this work we continue the investigation of undulator induced transparency (UIT). Besides the resonant right-hand polarized probe wave at electron-cyclotron frequency, we make the self-consistent consideration of left-hand polarized wave, which is also inevitably exited by the beating between probe and pump waves.

2. Basic equations

We will consider the propagation of two electromagnetic waves in magnetoactive plasma along the constant external magnetic field $\mathbf{H}=H\mathbf{z_0}$ and with the presence of the circular undulator magnetic field $\mathbf{B_w}=b_w\mathrm{Re}[\mathbf{e}_+\exp(ik_w z)]$:

$$\mathbf{A}_\perp=\mathrm{Re}\left\{\left[\mathbf{e}_+A_+(z)+\mathbf{e}_-A_-(z)\right]\exp(-i\omega t)\right\}. \qquad (1)$$

Here, A_- and A_+ are the vector potentials of the left-hand and right-hand polarized (probe) waves respectively, $\mathbf{A}_\perp$ is the total vector potential, $\mathbf{e}_\pm=2^{-1/2}(\mathbf{x_0}\pm i\mathbf{y_0})$ are the unity polarization vectors of the waves, k_w is the undulator wavenumber.

As it was shown before [2,3], in these conditions the longitudinal plasma oscillations are excited at the frequency ω. We will describe them by the potential:

$$\varphi_{||}=\mathrm{Re}\left\{\varphi_p(z)\exp(-i\omega t)\right\}. \qquad (2)$$

To investigate the propagation of the waves in undulator, we will use the hydrodynamic theory. The full set of equations has the following form:

$$\partial\mathbf{V}_\perp/\partial t+\omega_H\left[\mathbf{V}_\perp,\mathbf{z_0}\right]=(e/mc)\partial\mathbf{A}_\perp/\partial t-(e/mc)V_{||}\left[\mathbf{z_0},\mathbf{B_w}\right], \qquad (3)$$

$$\partial V_{||}/\partial t=(e/m)\partial\varphi_{||}/\partial z-(e/mc)(\mathbf{V}_\perp,\mathbf{B_w}), \qquad (4)$$

$$\left\{-\left(1/c^2\right)\partial^2/\partial t^2+\partial^2/\partial z^2\right\}\mathbf{A}_\perp=-(4\pi e/c)N_0\mathbf{V}_\perp, \qquad (5)$$

$$\partial/\partial t\left(\partial\varphi_{||}/\partial z\right)+4\pi eN_0V_{||}=0. \qquad (6)$$

Eqs. 3 and 4 are the Euler equations for transversal $V_\perp$ and longitudinal $V_{||}$ components of electron velocity, Eq. 5 is the wave equation and Eq. 6 describes the excitation of plasma oscillations. Here we used the Coulomb calibration. Also, $\omega_H=eH/mc$ is the electron gyrofrequency, N_0 is the unperturbed plasma density (N_0=const), e and m are the electron charge and mass, c is the velocity of light in vacuum. In this system only terms linear in $A_\perp$ are retained.

Let us introduce the complex amplitudes of corresponding variables:

$$\begin{aligned}\mathbf{V}_\perp&=\mathrm{Re}\left\{\mathbf{e}_+V_+\exp(-i\omega t)+\mathbf{e}_-V_-\exp(-i\omega t)\right\},\\ V_{||}&=\mathrm{Re}\left\{V_p\exp(-i\omega t)\right\}.\end{aligned} \qquad (7)$$

Then, only the terms with resonant frequencies are retained in Eqs. 3–6. This lead to the following system of coupled equations for right-hand polarized and left-hand polarized components of vector potential:

$$\begin{aligned}&\left\{-\left(c^2/\omega^2\right)d^2/dz^2-n_{0+}^2\right\}A_+=gA_-\exp(i2k_wz),\\ &\left\{-\left(c^2/\omega^2\right)d^2/dz^2-n_{0-}^2\right\}A_-=gA_+\exp(-i2k_wz),\end{aligned} \qquad (8)$$

while other variables in Eqs. 3–6 can be expressed via A_+ and A_-:

$$\begin{aligned}V_\pm&=(e/mc)\left\{\left(1-n_{0\pm}^2/v\right)A_\pm+(g/v)A_\mp\exp(\pm i2k_wz)\right\},\\ V_p&=\left(2igc/v\sqrt{u_w}\right)\left\{A_-\left(1-\sqrt{u}\right)\exp(in_wz)-A_+\left(1+\sqrt{u}\right)\exp(-in_wz)\right\}.\end{aligned} \qquad (9)$$

In system (8) n_{0+} is the refractive index of right-hand polarized wave if it propagates without the left-hand polarized one (i.e. A_-=0) and n_{0-} has the analogous meaning for left-hand polarized wave:

$$n_{0\pm}^2=1-v\frac{(1-v)\left(1\pm\sqrt{u}\right)-u_w/2}{(1-v)(1-u)-u_w}, \qquad (10)$$

and g is the coupling parameter:

$$g = \frac{1}{2}\frac{v u_w}{(1-v)(1-u)-u_w} \tag{11}$$

Also, $v=\omega_p^2/\omega^2$, $u=\omega_H^2/\omega^2$, $u_w=\omega_{Hw}^2/\omega^2$, where $\omega_p=(4\pi e^2 N_0/m)^{1/2}$ is the electron plasma frequency, $\omega_{Hw}=eb_w/2^{1/2}mc$ is the electron gyrofrequency, corresponding to the magnetic field in undulator.

Without the undulator (u_w=0), the expressions in (10) turn into the well-known equations of the refractive indexes of extraordinary and ordinary waves, propagating along the magnetic field in plasma [6]:

$$n_{0\pm}^2 = 1 - v\big/\left(1 \mp \sqrt{u}\right) \tag{12}$$

And when the undulator is present, only the dispersive curve of extraordinary wave changes significantly: the additional dispersive branch appears for $u_w \neq 0$. For ordinary wave the “nonlinear” dispersive curves are very close to the “linear” ones for $u_w \ll u$.

3. Propagation of the waves in UIT regime

After the substitution

$$A_{\pm}(z) = a_{\pm}(z)\exp(\pm i k_w z) \tag{13}$$

the system (8) turns into the system of ordinary differential equations for $a_{\pm}(z)$. The *exact* solution of this system can be found, using the standard methods:

$$A_{\pm} = A_{0\pm}\exp(i k_{\pm} z) \tag{14}$$

where, $k_{\pm}=k\pm k_w$ and the value of k should be found from the following dispersive relation:

$$\left\{(n+n_w)^2 - n_{0+}^2\right\}\left\{(n-n_w)^2 - n_{0-}^2\right\} - g^2 = 0 \tag{15}$$

Here, $n=ck/\omega$ and $n_w=ck_w/\omega$. The solution of Eq. 15 corresponds to four separate modes. If n_w=0, they are elliptically polarized extraordinary and ordinary waves in magnetoactive plasma and each wave can propagate along the $\mathbf{z_0}$ axis and in opposite direction. In this case the superposition of external magnetic field **H** and undulator field $\mathbf{B_w}$ turns into the constant magnetic field at the finite angle α (such as $\tan(\alpha)=B_w/H$) to the direction of propagation of the waves ($\mathbf{z_0}$ axis). And the expression for n turns into the well-known equations for the waves, propagating at the finite angle α relatively to the external magnetic field with amplitude $B_{tot}=(H^2+B_w^2)^{1/2}$ [6]:

$$n_{1,2}^2 = 1 - \frac{2v(1-v)}{2(1-v) - u_{tot}\sin^2\alpha \mp \sqrt{u_{tot}^2\sin^4\alpha + 4u_{tot}(1-v)^2\cos^2\alpha}} \tag{16}$$

Here, $u_{tot}=u+u_w$.

In general case ($n_w \neq 0$) the polarization ellipse of each normal wave rotates with the period of k_w/π. The general solution has the following form:

$$\mathbf{A}_{\perp} = \sum_{j=1}^{4} C_j \exp\left(i k_j z - i\omega t\right)\left[\mathbf{e}_+ \exp(i k_w z) + \mathbf{e}_- K_j \exp(-i k_w z)\right] \tag{17a}$$

Here, $k_j=(\omega/c)n_j$ and n_j is one of the four roots of Eq. 15, the value of K_j determines, which of the waves prevails:

$$K_j = \left(n_j + n_w\right)^2 - n_{0+}^2\big/g = \left(\left(n_j - n_w\right)^2 - n_{0-}^2\big/g\right)^{-1} \tag{18}$$

The ratio between the axes of the polarization ellipse and its orientation in the space are determined by the parameter:

$$A_x\big/A_y = -i\left\{1 + K_j\exp(-i2k_w z)\right\}\big/\left\{1 - K_j\exp(-i2k_w z)\right\} \tag{19}$$

Obviously, the solution (17a) can also be represented as the superposition of 4 pairs of the waves, where each pair consists of right-hand polarized and left-hand polarized waves with different wavenumbers:

$$\mathbf{A}_{\perp} = \sum_{j=1}^{4} C_j \exp(-i\omega t)\left[\mathbf{e}_+ \exp(i k_{j+} z) + \mathbf{e}_- K_j \exp(-i k_{j-} z)\right] \tag{17b}$$

Here, $k_{j\pm}=k_j\pm k_w$.

4. Mode conversion

Fig. 1 shows the behavior of the refractive index n in inhomogeneous plasma for different fixed values of u. The regions, where A_+ or A_- prevails are marked appropriately.

The component A_+ predominates in the field (17a), if the condition $|k_+^2 - \omega^2 n_{0+}^2/c^2| \ll |\omega g/c|^2$ is satisfied and for the predomination of A_- the fulfillment of condition $|k_-^2 - \omega^2 n_{0-}^2/c^2| \ll |\omega g/c|^2$ is necessary. If two certain roots n_i and $n_{j\neq i}$ become close to each other, such that $|n_i - n_j| \ll |g|$ (see also Fig. 1), then the right-hand polarized wave, corresponding to the solution n_i and the left-hand polarized wave, corresponding to the solution n_j, became coupled via the expression $|k_{i+} - k_{j-} - 2k_w| \ll (\omega/c)|g|$. As it can be clearly seen from Eq. 8, this happens in the area of parameters, where:

$$\left|n_{0+} - n_{0-} - 2n_w\right| \ll |g|\,. \tag{20}$$

In this region there is the possibility of mode conversion in inhomogeneous plasma, i.e. the wave passes from one dispersive branch to another. It is important to note, that this is the conversion of one two-wave mode in (17b) to another.

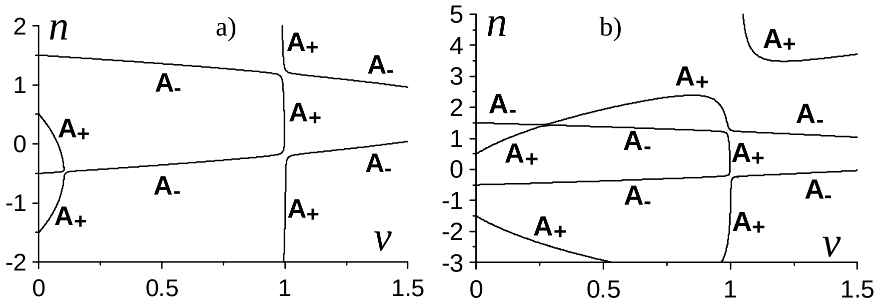

Fig. 1. Dependency $n(v)$ in UIT regime. n_w=0.5, $u_w=6\cdot10^{-3}$, u=0.8 (a), u=1.2 (b).

The conversion efficiency is in the order of the exp(–δ) where [6]:

$$\delta = \left| \frac{\omega}{c(dv/dz)} \oint_L \frac{n_1 - n_2}{2} dv \right| , \qquad (21)$$

n_1 and n_2 are the values of refractive index n on two dispersive branches in the coupling region, L is contour, surrounding the branching points of function $G(v)=n_1(v)–n_2(v)$.

Since the expressions for n_1 and n_2 can not be found analytically from Eq. 15 for $n_w\neq 0$, we will approximate the function $G(v)$ in the coupling region by two hyperbolas by expanding the Eq. 15 in terms of the small parameters $\delta n=n–n_0$ and $\delta v=v–v_0$ and dropping the terms of order δn^3, δv^3 and higher. Here, v_0 and n_0 are the values of corresponding parameters in the point of intersection between curves $n_{0+}–n_w$ and $n_{0–}+n_w$. The resulted expression for δ is rather cumbersome, but for the coupling region near the point v=1 it can be further simplified:

$$\delta \approx \frac{2\pi\omega}{c|dv/dz|} \frac{1+\sqrt{u}}{8(N_0 - n_w)} \frac{u_w^2}{(1-v_0)(1-u)-u_w} \sim \frac{u_w^2}{(1-v_0)(1-u)-u_w} L . \qquad (22)$$

where $L=|dv/dz|^{-1}$ is the inhomogeneity scale of the plasma.

The incident right-hand polarized wave should be converted into the left-had polarized with minimal efficiency, for the most effective excitation of electrostatic oscillations (see Eq. 9). Requiring the small efficiency: δ≥1 (which corresponds to the conversion less than 50%), one can find the inhomogeneity scale L. For example, the inhomogeneity scale should be greater than 5(c/ω) for the following parameters: n_w=1.6, u_w=0.04, u=1.

5. Proper profiles of plasma density and external magnetic field

The investigations above were made for infinite plasma. However, to put the real experiment into practice, it is necessary to consider the propagation of the waves through the finite plasma region. Let us assume, that plasma density never exceeds the critical one (v<1). As it can be seen from expression for n_{0+}, for propagation of probe wave ($n_{0+}^2>0$) the following condition should be satisfied:

$$(1-v)(1-u)-u_w < 0 , \qquad (23)$$

which means, that the magnetic field should increase near the bounds of plasma region (in other words it should have the magnetic trap configuration).

The propagation of incident right-hand polarized radiation through the plasma slab was investigated in details using the particle-in-cell simulations. The results of these calculations, demonstrating the effect of UIT are shown at Fig. 2. It can be seen, that the incident probe wave is transmitted through the plasma region at electron-cyclotron frequency and electrostatic oscillations are effectively excited. About 30% of incident energy is converted into the nonresonant component, which is in the agreement with estimations, following from Eq. 22.

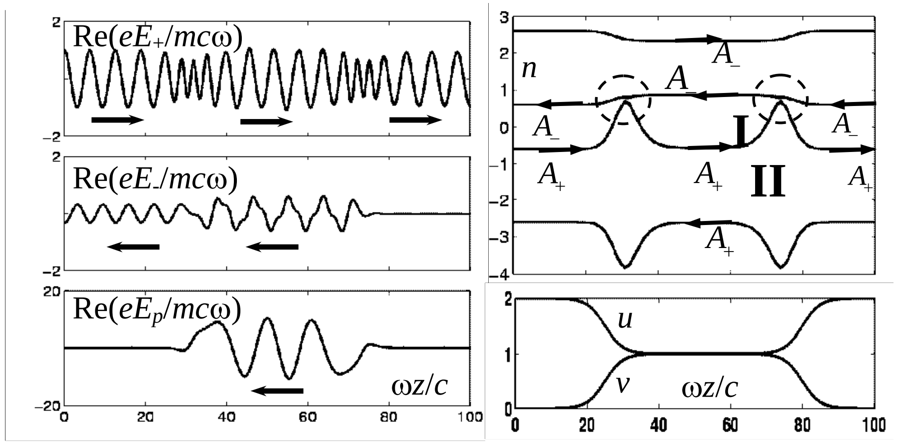

Fig. 2. Numerical calculations, demonstrating the UIT effect. n_w=1.6, u_w=0.04, u=1, v=0.99 (in the center of plasma). The arrows show the direction of propagation for the waves. The circles denote the regions of mode conversion.

6. Conclusion

In this work we theoretically investigated the self-consistent structure of normal waves in the regime of UIT for the longitudinal propagation of the waves in cold magnetized plasma. It is shown, that there is the possibility of mode conversion for these waves during the propagation in inhomogeneous plasma. The efficiency of mode conversion is estimated.

Numerical simulations were used for modelling this effect and the detailed comparison of the results with the theory is performed. Various profiles of the plasma density and external axial magnetic field are studied. The proper selection of undulator wavelength combined with judicious choice of axial magnetic field profile enables microwaves to penetrate plasma with realistic (smooth) density profiles at moderate level of the undulator field, suppressing the strong resonant absorption at the cyclotron frequency.

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